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The effect of kinetic energy change on flow in gas pipelines

The significance of the acceleration term in pressure drop calculations is investigated

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Engineers often seek simple design tools that can be utilized for quick and reliable calculations. This approach has led to emergence of many simplified or integrated forms of pressure drop equations that are widely used for gas pipeline design. In most of such analytical expressions the effect of kinetic energy on the total pressure drop is, however, neglected and may result in substantial errors. In this article, the significance of the acceleration term in pressure drop calculations of gas pipelines is investigated. A simple procedure is introduced that provides an estimate of the pressure drop contribution due to the kinetic energy change to the total pressure drop along the pipeline at different operating conditions. The procedure is then applied to a number of case studies to determine the operating region above which the effect of kinetic energy change on the gas flow is relatively significant.

The steady-state momentum balance around a differential control volume of a pipe segment is:

$$\frac{dP}{dL} = -\frac{\rho g \sin\theta}{g_c} - \frac{24\rho f u^2}{g_c D_i} - \frac{\rho u}{\alpha g_c}\frac{du}{dL} \tag{1}$$

It is evident that three phenomena: elevation, friction and acceleration, comprise the pressure drop in a pipeline. In Eq. 1, the correction factor, α, represents the gas velocity profile variation over the pipe cross-sectional area. This correction factor depends on the velocity profile and typically varies from 0.75 for laminar flow to about 1.0 for fully developed turbulent flow;[1] Aziz[2] suggested that a value of 0.9 provides a reasonable estimate for practical pipeline modeling. To solve Eq. 1, the gas density, ρ, and friction factor, f, should also be computed. The gas density can be obtained from an appropriately chosen equation of state (EOS), whereas the friction factor is normally calculated from the Colebrook and White equation:[3]

$$f^{0.5} = -4\log\left(\frac{\varepsilon}{3.71 D_i} + \frac{1.255}{f^{0.5} Re}\right) \tag{2}$$

In a preceding study,[4] the contribution of hydraulic term, i.e., gravitational energy, to the total pressure drop in gas distribution networks was investigated at various gas flowrates and pipe inclinations. In this article, the pressure gradient due to the kinetic energy change or convective acceleration, i.e., $-\rho u/\alpha g_c \times du/dL$ has received particular attention. In principle, pressure drop due to the acceleration occurs in all transient flow conditions; but it is zero for incompressible flow in pipelines with constant cross-sectional area. For any flow condition at which velocity change takes place, pressure drop will occur in the direction of the velocity increase.

A simplified analytical solution of Eq. 1 is its integrated form that represents the general form of the flow equation as:

$$Q = \left(\frac{\pi}{8}\right)\left(\frac{T_b}{P_b}\right)\left(\frac{R g_c}{28.97}\right)^{0.5} \frac{D_i^{2.5}}{\sqrt{f Z_{ave} T_{ave} \gamma_g}} \times \left(\frac{P_1^2 - e^{S_k} P_2^2}{L}\frac{S}{e^{S_k} - 1}\right)^{0.5} \tag{3}$$

wherein:

$$S_k = \frac{S\left[1 - \left(D_i/2\alpha f L\right)\mathrm{Ln}\left(P_2/P_1\right)\right]}{1 + \left(D_i/4\alpha f L\right)S},$$

$$S = \frac{57.94\gamma_g g}{Z_{ave} R T_{ave} g_c} L\sin\theta \tag{4}$$

Eq. 3 accounts for the effects of elevation, friction and kinetic energy change on the total pressure drop along the pipeline. Hence, this equation is the most comprehensive form of the integrated flow equation published so far. By neglecting the acceleration term, Eq. 3 can be simplified to:

$$Q = \left(\frac{\pi}{8}\right)\left(\frac{T_b}{P_b}\right)\left(\frac{R g_c}{28.97}\right)^{0.5} \frac{D_i^{2.5}}{\sqrt{f Z_{ave} T_{ave} \gamma_g L}} \times \left[\left(P_1^2 - P_2^2\right) - \frac{57.94\gamma_g P_{ave}^2 g}{R T Z_{ave} g_c}\Delta H\right]^{0.5} \tag{5}$$

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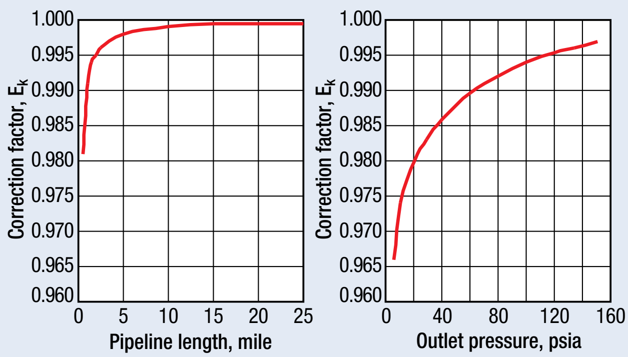

FIG. 1 Flow correction factor, E_k, for a gas pipeline.[5]

In an extensive research on gas pipelines, Ouyang and Aziz[5] studied the effect of kinetic energy change on gas flowrates in a gas pipeline with specifications given in Table 1. Variations of the correction factor, $E_k = Q$ (from Eq. 3)/Q (from Eq. 5), with respect to the pipeline length and outlet pressure are shown in Fig. 1. These results revealed that in certain cases, the error caused by neglecting the kinetic energy change can reach about 3.37%. Young[6] stated that this error may reach up to 9%, whereas Tian and Adewumi[7] reported 28% and 43% errors in outlet pressure calculations for special gas flow problems. In the latter study pressure drops due to the kinetic energy change were 6.8% and 5.0% of the total pressure drops. These conditions are, however, not likely to be encountered in practice. It is worth noting that in the studies carried out by Young[6] and Tian and Adewumi[7] the correction factor, α, was ignored. In the following, a simple procedure is introduced to estimate the acceleration contribution to the total pressure drop in a gas pipeline.

The continuity equation for a differential control volume of a gas pipeline with constant cross-sectional area is:

$$\frac{\mathrm{d}(\rho u)}{\mathrm{d}L} = 0 \quad \text{or} \quad \rho\frac{\mathrm{d}u}{\mathrm{d}L} + u\frac{\mathrm{d}\rho}{\mathrm{d}L} = 0 \tag{6}$$

TABLE 1. Test data for a gas pipeline

Pipe ID, in.	4.0
Pipe roughness, μ in.	600.0
Pipeline length, mile	0.568
Inclination angle	0
Gas gravity	0.75
Gas viscosity, cP	0.018
Pseudo-critical pressure, psia	661.0
Pseudo-critical temperature, R	411.0
Average temperature, °F	545.0
Inlet pressure, psia	200.0
Outlet pressure, psia	30.0

Assuming that the gas temperature is constant along the pipe, which is usually a valid assumption for gas pipelines, rearranging Eq. 6 yields:

$$-\frac{\rho u}{\alpha g_c}\frac{\mathrm{d}u}{\mathrm{d}L} = \frac{u^2}{\alpha g_c}\frac{\mathrm{d}\rho}{\mathrm{d}L} \tag{7}$$

The ratio of the pressure drop due to the acceleration to the total pressure drop can now be defined as:

$$\psi = \frac{\dfrac{u^2\mathrm{d}\rho}{\alpha g_c \mathrm{d}L}}{\dfrac{\mathrm{d}P}{\mathrm{d}L}} = \frac{u^2}{g_c}\left(\frac{\mathrm{d}\rho}{\mathrm{d}P}\right)_T =$$

$$\left(\frac{4\rho_b}{\pi}\right)^2\left(\frac{Q}{600D_i^2}\right)^2\frac{1}{144\alpha g_c\rho^2\left(\dfrac{\mathrm{d}P}{\mathrm{d}\rho}\right)_T} \tag{8}$$

where Q is the gas volumetric flowrate at standard condition, i.e., 14.7 psia and 60°F, ρ_b, is the gas density at standard condition, D_i, is the pipe inner diameter and ρ is the gas density at the operating temperature and pressure.

Dranchuk and Abu-Kassem[8] proposed an EOS for the natural gas compressibility factor that is in fact an approximation to Standing and Katz's graphical representation of the compressibility factor for sweet natural gases with molecular weights less than 40.[9] This EOS can be used for determining the gas density:

$$Z = 1 + \left[A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5}\right]\rho_r + \left[A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}\right]\rho_r^2 - A_9\left[\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}\right]\times \rho_r^5 + A_{10}\left(1 + A_{11}\rho_r^2\right)\frac{\rho_r^2}{T_{pr}^3}\exp\left[-A_{11}\rho_r^2\right] \tag{9}$$

The coefficients of Eq. 9 are listed in Table 2. Eq. 9 can be rearranged by substituting $28.95\gamma_g P/\rho RT$ in place of Z to derive the following explicit expression for $\rho^2 \times (\mathrm{d}P/\mathrm{d}\rho)_T$:

$$\rho^2\left(\frac{\partial P}{\partial \rho}\right)_T = RT\rho^2 + \frac{2RT\rho^3}{\rho_c}\left[A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5}\right] + \frac{3RT\rho^4}{\rho_c^2}\left[A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}\right] - \frac{6RT\rho^7}{\rho_c^5}A_9\left[\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}\right] + \frac{RT\rho^4}{T_{pr}^3\rho_c^4}A_{10}\left(3\rho_c^2 + 3A_{11}\rho^2 - \frac{2A_{11}^2\rho^4}{\rho_c^2}\right)\exp\left[-A_{11}\frac{\rho^2}{\rho_c^2}\right] \tag{10}$$

wherein:

$$\rho_c = \frac{28.95\gamma_g P_c}{Z_c R T_c} \quad (11)$$

The critical compressibility factor, Z_c, is obtained from the Standing and Katz graph to be 0.27. Furthermore, the critical properties of natural gas can be estimated from the empirical correlations proposed by Standing:[10]

$$T_c = 168 + 325\gamma_g - 12.5\gamma_g^2 \quad (12)$$

$$P_c = 677 + 15.0\gamma_g - 37.5\gamma_g^2 \quad (13)$$

Given that the term $1/[\rho^2 \times (dP/d\rho)]_T$ is determined from Eqs. 10 to 13, the contribution of acceleration to the total pressure drop is simply estimated from Eq. 8 for various Q/D_i^2 values. This procedure can be used to determine when this contribution is significant and, therefore, cannot be neglected in pressure drop calculations. It should be noted that for further convenience, the dependence of gas density on the operating temperature and pressure, as well as the gas specific gravity, can also be estimated from the Beggs-Brill equation:[11]

$$Z = A + \frac{(1-A)}{e^B} + CP_{pr}^D$$

$$A = 1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101$$

$$B = (0.62 - 0.23T_{pr})P_{pr} + \left[\frac{0.066}{(T_{pr} - 0.86)} - 0.037\right]P_{pr}^2 + \frac{0.32}{10^{9(T_{pr}-1)}}P_{pr}^6 \quad (14)$$

$$C = 0.132 - 0.32\log(T_{pr})$$

$$D = 10^{(0.3016 - 0.49T_{pr} + 0.1824T_{pr}^2)}$$

Fig. 2 shows the variation of ψ with respect to the operating pressure at different Q/D_i^2 values when the correction factor, α, is assumed to be 1.0. The gas temperature and specific gravity are 520.0 R and 0.65, respectively. As can be seen, the contribution of pressure drop due to the acceleration to the total pressure drop is insignificant at high operating pressures. As the operating pressure decreases, ψ tends to rise. Hence, the effect of kinetic energy change on the pressure drop becomes more profound when the operating pressure of a gas pipeline drops, e.g., steep slopes in inclined terrains, near compressor stations, etc. Fig. 2 reveals that for Q/D_i^2 values below 2×10^6, ψ becomes negligible at operating pressures higher than 150 psia. It is self-evident from Eq. 8 that the effect of kinetic energy change on the total pressure drop is somewhat more significant at lower velocity profile correction factors.

The effect of kinetic energy change on pressure drop in gas distribution networks is also studied. These networks handle lower gas flowrates and their operating pressure typically falls in the range of 50 to 150 psia. Fig. 3 demonstrates the variation of ψ in gas distribution networks where the gas temperature and specific gravity are 520.0 R and 0.65, respectively. It is apparent that the effect of kinetic energy change in such networks is negligible.

To verify the validity of assuming constant temperature in deriving Eq. 8, another case study was realized in which the effect of temperature variations on the pressure drop calculation was also taken into account. The algorithm proposed and validated by Abdolahi et al.[12] was employed to simulate flow in gas pipelines and compute the ratio of pressure drop due to the acceleration to the total pressure drop. Using the Peng-Robinson EOS[13] and Lucas method[14] to determine the thermo-physical properties and viscosity respectively, the effect of kinetic energy change on the total pressure drop of a gas flow with the specifications given in Table 3 was computed when α = 1. Fig. 4 shows the simula-

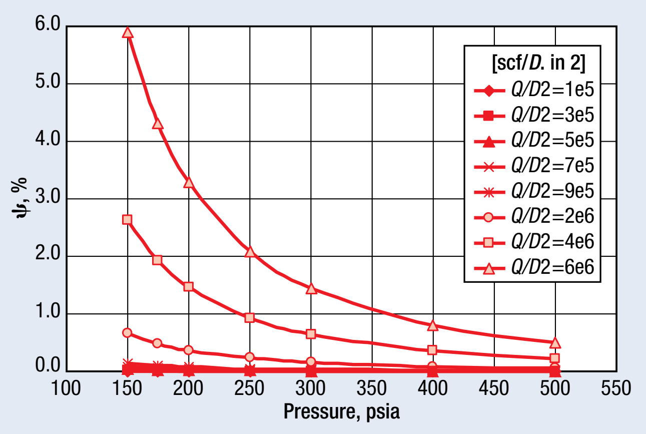

FIG. 2 Ratio of the pressure drop due to the kinetic energy change to the total pressure drop in gas pipelines.

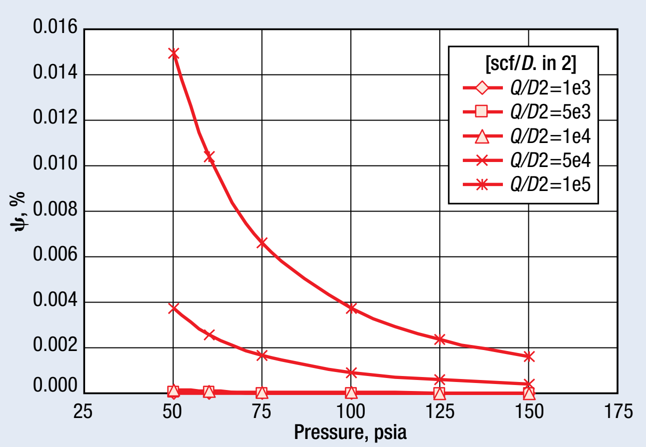

FIG. 3 Ratio of the pressure drop due to kinetic energy to the total pressure drop in gas distribution networks.

TABLE 2. Coefficients of Dranchuk and Abu-Kassem EOS[8]

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
0.3265	–1.07	–0.5339	0.01569	–0.05165	0.5475	–0.7361	0.1844	0.1056	0.6134	0.721

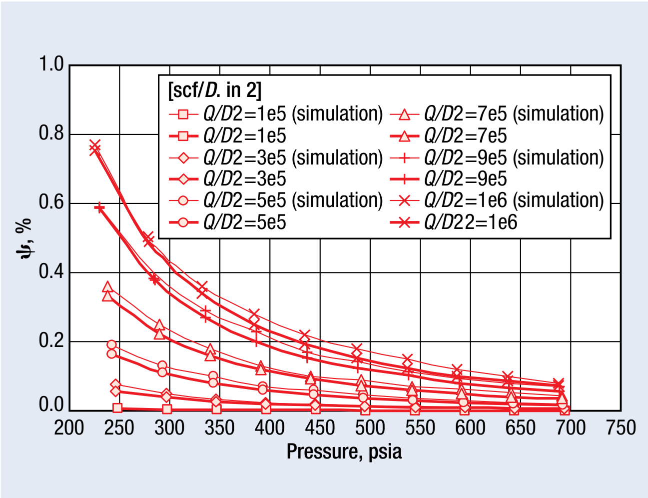

FIG. 4 Simulation results show close agreement with the estimates obtained using the simplified procedure.

TABLE 3. Test data for a gas pipeline used for simulations

Gas composition, mole percent	$C_1 = 98.30$, $C_2 = 0.682$, $C_3 = 0.070$, n-$C_4 = 0.035$, i-$C_4 = 0.170$, n-$C_5 = 0.023$, i-$C_5 = 0.017$, $C_6^+ = 0.204$, $CO_2 = 0$, $N_2 = 0.499$
Inclination angle, degrees	5.0
Pipe roughness, μ in.	25.4
Burial depth, ft	4.0
Soil thermal conductivity, Btu/ft^2 hr °F	1.0
Ground temperature, R	560.0
Inlet temperature, R	562.2

tion results compared to the estimates obtained by means of the simplified procedure proposed in this study. As can be seen, the estimates provided by Eq. 8 are in good agreement with the simulation results suggesting the validity of the underlying assumptions of the proposed method. **HP**

NOMENCLATURE

- A Beggs-Brill equation coefficient
- $A_{1,\ldots,11}$ Dranchuk and Abu-Kassem EOS coefficients
- B Beggs-Brill equation coefficient
- C Beggs-Brill equation coefficient
- D Beggs-Brill equation coefficient
- D_i pipe inner diameter, in.
- f Fanning friction factor
- g gravitational acceleration, ft/s^2
- g_c conversion factor, 32.174, lb ft/s^{2}lb$_f$
- H elevation, ft
- L pipe length, mile
- P pressure, psia
- P_{ave} average pressure, psia
- P_b pressure at standard conditions, usually 14.696 psia
- P_c critical pressure, psia
- P_{pr} pseudo reduced pressure
- Q gas flowrate, scf/d
- R universal gas constant, psia ft^3/lb mol R)
- Re Reynolds number
- S dimensionless elevation factor with the kinetic energy change ignored
- S_k dimensionless elevation and kinetic energy change factor
- T gas temperature, R
- T_{ave} gas average temperature, R
- T_b temperature at standard condition, usually 520 R
- T_c critical temperature, R
- T_{pr} pseudo reduced temperature
- u gas velocity, ft/s
- Z compressibility factor
- Z_{ave} average compressibility
- Z_c critical compressibility factor
- α correction factor to compensate for variations in the velocity profile over the pipe cross-section
- ε absolute pipe roughness, in.
- γ_g gas specific gravity, which is defined as the ratio of density of dry air with both at standard temperature and pressure
- μ_g gas viscosity, cP
- θ inclination angle with horizontal line, degrees
- ρ gas density, lb/ft^3
- ρ_b gas density at standard condition, lb/ft^3
- Δ difference

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