Stochastic Model Predictive Control with Integrated Experiment Design for Nonlinear Systems

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Abstract: The performance of predictive control strategies often degrades over time due to growing plant-model mismatch. Closed-loop performance restoration typically requires some form of model maintenance to reduce model uncertainty. This paper presents a stochastic predictive control approach with integrated experiment design for nonlinear systems with probabilistic modeling uncertainties. The integration of predictive control with experiment design enables enhancing the information content of closed-loop data for online model adaption. The presented approach considers control-oriented experiment design to ensure adequate model adaptation (in probability) in terms of an admissible control performance level. The stochastic optimal control approach is demonstrated on a continuous bioreactor case study.

Keywords: Dual predictive control, Stochastic optimal control, Control-oriented model adaptation

1. INTRODUCTION

Model predictive control (MPC) is widely used for advanced control of complex systems due to its ability to cope with multivariable system dynamics, system constraints, and competing control objectives (Morari and Lee, 1999). A key challenge in MPC applications arises from the imperfect knowledge of system dynamics. Plant-model mismatch can largely restrict the MPC performance. Thus, some form of model maintenance must often be performed in MPC applications to ensure adequate closed-loop performance (e.g., see (Mesbah et al., 2015)).

The ability to generate input-output data sets with high information content is crucial for system identification. Optimal experiment design enables systematic excitation of system dynamics to obtain informative data sets for model structure and/or parameter identification (e.g., (Pronzato, 2008)). However, model-based input design for system identification and model-based control intrinsically seek conflicting objectives. The former aims at exciting the system dynamics to maximize the information content of the input-output data, whereas in control the primary objective is typically to suppress disturbances and perturbations. For linear systems, model-based control strategies have recently been proposed that integrate experiment design with predictive control (Marafioti et al., 2013; Larsson et al., 2015; Heirung et al., 2015). In these control strategies, some measure of the information content of system outputs is incorporated into the optimal control problem. Thus, the designed control inputs will have some form of *dual effect* to enable generating informative closed-loop data for model uncertainty reduction (Wittenmark, 1995). Larsson et al. (2013) presented a MPC approach with integrated control-oriented experiment design, where

the intended control application of the model is explicitly accounted for in input design (Hjalmarsson, 2005).

This paper addresses the problem of probabilistic model uncertainty handling in the context of stochastic predictive control (Mesbah, 2016) of nonlinear systems. A stochastic MPC approach with integrated control-oriented experiment design (iX-SNMPC) is presented that aims at not only regulating the system dynamics, but also enhancing the information content of the closed-loop data for control-oriented model identification. The closed-loop data is used for estimating the (posterior) probability distribution of model parameters at each sampling instant in order to reduce model uncertainty. An *experiment design chance constraint* is incorporated into the stochastic optimal control problem to ensure, in probability, that the identified model satisfies an admissible control performance level (Rojas et al., 2011). In contrast to standard experiment design approaches that rely on the best estimate of model parameters (Gevers, 2005), the iX-SNMPC approach considers the full probability distribution of the uncertain model parameters in the input design problem. The performance of the proposed approach is evaluated using a continuous bioreactor benchmark (Agrawal et al., 1989).

Notation.

$\mathbb{N}$ denotes the set of natural numbers; $\mathbb{N}_0 = \mathbb{N} \cup 0$. $\mathsf{P}(\cdot)$ denotes the probability distribution function (pdf) of a stochastic variable. $\mathsf{P}(\cdot|z)$ denotes the pdf of a stochastic variable conditioned on z. $\mathcal{N}(\mu, \Sigma)$ denotes a Gaussian distribution with mean μ and covariance Σ; $\mathcal{N}(x; \mu, \Sigma)$ denotes the value of the pdf at x. $\mathbf{Pr}[\cdot]$ denotes probability. $\mathbf{E}[\cdot]$ and $\mathbf{Var}[\cdot]$ denote expected value and variance, respectively. $\det(\cdot)$ and $\lambda_i(\cdot)$ denote the determinant and the ith eigenvalue of a matrix, respectively.

2. PROBLEM FORMULATION

Consider a discrete-time, nonlinear system

$$x(t) = f\Big(x(t-1), u(t-1), \theta_0\Big), \quad x(0) = x_0, \qquad (1)$$

where t denotes the time index; $x \in \mathbb{R}^n$ denotes the system states with initial conditions x_0; $u \in \mathbb{R}^m$ denotes the inputs; $\theta_0 \in \mathbb{R}^p$ denotes the true system parameters; and f denotes the nonlinear state dynamics. The function f is assumed to be of polynomial form. Nonpolynomial functions can be transformed to polynomial-in-states form if f is analytic with respect to x and separable with respect to x and θ_0 (Papachristodoulou and Prajna, 2005).

The model used for describing system (1) is subject to probabilistic uncertainty arising from imperfect knowledge of the system parameters θ_0. According to the prediction error identification framework, the identified model parameters $\{\hat{\theta}_i\}_{i=1}^{p}$ are independently normally distributed, that is $\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_\theta)$ with $\mathcal{P}_\theta$ being the parameter covariance matrix (Ljung, 1999).

The system inputs are subject to hard constraints

$$u(t) \in \mathbb{U} \triangleq \{H_u u(t) \le d_u\},$$

where $H_u \in \mathbb{R}^{s \times m}$, $d_u \in \mathbb{R}^s$, and $s \in \mathbb{N}$ is the number of input constraints. The states are constrained as

$$\mathbb{X}_i \triangleq \{x_i(t) \in \mathbb{R} \mid c_i x_i + d_i \le 0\}$$

with $c_i \in \mathbb{R}$ and $d_i \in \mathbb{R}$ being constants. To effectively handle state constraints in the presence of the (unbounded) probabilistic model uncertainty, the state constraints are replaced with individual chance constraints

$$\mathbf{Pr}\left[x_i(t) \in \mathbb{X}_i\right] \ge \beta_i, \quad \forall\, i = 1, \dots, n,$$

where $\beta_i \in (0, 1)$ is the lower bound for the probability level that each state chance constraint must be satisfied.

The goal of this work is to develop a stochastic predictive control approach with integrated control-oriented experiment design capability for the nonlinear system (1). The optimal control inputs should serve two purposes: (i) regulate the system dynamics in terms of the defined control objectives, and (ii) excite the system dynamics such that the closed-loop data can be used for identifying a model that ensures achieving a prescribed control performance in probability (due to model uncertainty). This work only addresses parametric model uncertainty, that is, the uncertainty associated with $\hat{\theta}$. In what follows, a model of system (1) is represented by the parameter vector $\hat{\theta} \in \mathbb{R}^p$.

Control-oriented experiment design involves defining an application set $\Psi_{\text{app}}(\theta)$, which consists of all system models that result in admissible control performance. The application set is defined by

$$\Psi_{\text{app}}(\theta) = \left\{\theta : V_{\text{app}}(\theta) \le \frac{1}{\gamma}\right\},$$

where $V_{\text{app}}(\theta)$ is an application cost function that quantifies the control performance degradation due to plant-model mismatch (i.e., the discrepancy between θ_0 and θ). $\gamma > 0$ is an application specific, user-defined bound based on which the control-oriented adequacy of a model is defined. An identified model would be considered adequate if $V_{\text{app}}(\theta) < 1/\gamma$.

The objective of control-oriented experiment design is to identify a system model that lies in the application set $\Psi_{\text{app}}(\theta)$, that is, $\hat{\theta} \in \Psi_{\text{app}}(\theta)$ (Rojas et al., 2011). A key challenge in experiment design arises from the fact that the knowledge of model parameters $\hat{\theta}$, on which the experiment design relies, is subject to uncertainty (i.e., $\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_\theta)$). Hence, due to the probabilistic uncertainty associated with the identified parameters $\hat{\theta}$, the control-oriented input design requirement $\hat{\theta} \in \Psi_{\text{app}}(\theta)$ can only be satisfied in probability

$$\mathbf{Pr}[\hat{\theta} \in \Psi_{\text{app}}(\theta)] \ge 1 - \varepsilon, \qquad (2)$$

where $\varepsilon \in (0, 1)$.

The chance constraint (2) represents the requirement for control-oriented experiment design, and is used to formulate the proposed iX-SNMPC approach. Under full-state feedback, the iX-SNMPC approach involves solving the following stochastic optimal control problem at every sampling instant t_k

$$\pi^* \triangleq \arg \min_{\pi}\; J[\pi, x(t_k)] \qquad (3a)$$

$$\text{s.t.: } \bar{x}(t) = f(\bar{x}(t-1), u(t-1), \hat{\theta}), \qquad (3b)$$
$$\forall\, t \in [t_k \quad t_{k+N}]$$

$$\mathbf{Pr}\left[\bar{x}_i(t) \in \mathbb{X}_i\right] \ge \beta_i, \qquad \forall\, i = 1, \dots, n \qquad (3c)$$
$$\forall\, t \in [t_k \quad t_{k+N}]$$

$$u(t) \in \mathbb{U}, \qquad \forall\, t \in [t_k \quad t_{k+N}] \qquad (3d)$$

$$\mathbf{Pr}[\hat{\theta} \in \Psi_{\text{app}}(\theta)] \ge 1 - \varepsilon \qquad (3e)$$

$$\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_\theta) \qquad (3f)$$

$$\bar{x}(t_k) = x(t_k), \qquad (3g)$$

where $\pi \triangleq [u(t_k), \dots, u(t_{k+N})]^\top$ denotes the control policy; N is the prediction horizon; and $J[\pi, x(t_k)]$ denotes a general cost function that can be defined to shape the pdf of states either in terms of their full distributions (e.g., (Buehler et al., 2016)) or their statistics (e.g., (Mesbah et al., 2014)). The optimal solution to the stochastic optimal control problem (3) is denoted by π^*. The iX-SNMPC is implemented in a receding-horizon manner, which implies that at time t_k only the optimal inputs $u^*(t_k)$ are applied to system (1). The stochastic optimal control problem (3) is subject to state chance constraints to ensure state constraint satisfaction in the presence of probabilistic model uncertainty. In addition, the experiment design chance constraint (3e) ensures that a system model identified from the closed-loop data can guarantee an admissible control performance in a probabilistic sense.

There are several challenges associated with deriving a tractable surrogate for (3). The first challenge is to define a suitable control-oriented application set $\Psi_{\text{app}}(\theta)$. The application set requires the knowledge of the true system. However, the true system is generally unknown. As is a common practice in experiment design, a system model based on the best estimates of parameters can be used to construct the application set (Larsson, 2011). This approach, however, cannot account for the effects of model uncertainty. In this paper, the probabilistic description of parametric uncertainties is considered in evaluating the experiment design chance constraint (3e). The generalized polynomial chaos (gPC) framework (Xiu and Karniadakis, 2002) is used for uncertainty propagation. The second challenge in solving (3) lies in obtaining a tractable,

deterministic surrogate for (3e). In this work, the Chernoff relaxation is used to derive an analytic expression for (3e) (Nemirovski and Shapiro, 2006).

3. IX-SNMPC APPROACH

This section describes the methods adopted to obtain a computationally tractable surrogate for (3).

3.1 Application Cost Function

The application cost function $V_{app}(\theta)$ provides a measure for the control performance degradation due to plant-model mismatch. $V_{app}(\theta)$ is defined in terms of the difference between the measured system states when the controller is designed using the true parameter values θ_0 and when the controller is designed using perturbed parameters θ (Ebadat et al., 2014)

$$V_{app}(\theta) \triangleq \frac{1}{M} \sum_{t=1}^{M} \| x(t,\theta_0) - x(t,\theta) \|^2, \qquad (4)$$

where M is the number of measurements. The application cost function $V_{app}(\theta)$ has its minimum value at 0 when $\theta = \theta_0$. This implies that $V_{app}(\theta_0) = V'_{app}(\theta_0) = 0$.

To obtain a convex surrogate for the experiment design constraint (2), this paper considers a second-order approximation of the cost function $V_{app}(\theta)$ (Larsson, 2011)

$$
\begin{aligned}
V_{app}(\theta) &\approx V_{app}(\theta_0) + V'_{app}(\theta_0)(\theta - \theta_0) \\
&+ \frac{1}{2}(\theta - \theta_0)^\top V''_{app}(\theta_0)(\theta - \theta_0) \\
&= \frac{1}{2}(\theta - \theta_0)^\top V''_{app}(\theta_0)(\theta - \theta_0),
\end{aligned}
\qquad (5)
$$

where the second derivative $V''_{app}(\theta_0)$ can be approximated by the Hessian matrix of the system states (Ebadat et al., 2014). Notice that the application cost function and its derivatives are dependent on the true parameters θ_0, which are unknown. In this work, the pdfs of the estimated parameters $\hat{\theta}$ are used to represent the unknown true parameters θ_0.

Using the approximation (5), the control-oriented experiment design chance constraint (2) takes the form

$$\mathbf{Pr}\left[\frac{1}{2}(\hat{\theta} - \theta_0)^\top V''_{app}(\hat{\theta} - \theta_0) \geq \frac{1}{\gamma}\right] \leq \varepsilon. \qquad (6)$$

Next, a convex relaxation for (6) is discussed.

3.2 Approximation of the Experiment Design Chance Constraint

Inspired by the analysis provided in (Rojas et al., 2011) on chance constrained input design, this work uses the Chernoff relaxation (Nemirovski and Shapiro, 2006) to obtain a computationally tractable surrogate for (6). The Chernoff bound on (6) is derived as

$$\mathbf{Pr}\left[\frac{1}{2}(\hat{\theta} - \theta_0)^\top V''_{app}(\hat{\theta} - \theta_0) \geq \frac{1}{\gamma}\right]$$

$$= \mathbf{Pr}\left[z^\top \mathcal{P}(t)^{-\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{-\frac{1}{2}} z - \frac{2}{\gamma} \geq 0\right] \qquad (7a)$$

$$= \mathbf{Pr}\left[\exp\left(\sum_{i=1}^{p} \frac{1}{\rho} \lambda_i \left(\mathcal{P}(t)^{-\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{-\frac{1}{2}}\right) \tilde{z}_i^2 - \frac{2}{\gamma\rho}\right) \geq 1\right] \qquad (7b)$$

$$\leq \mathbf{E}\left[\exp\left(\sum_{i=1}^{p} \frac{1}{\rho} \lambda_i \left(\mathcal{P}(t)^{-\top/2} V''_{app} \mathcal{P}(t)^{-1/2}\right) \tilde{z}_i^2 - \frac{2}{\gamma\rho}\right)\right], \qquad (7c)$$

where $z \triangleq \mathcal{P}(t)^{-1/2}(\hat{\theta} - \theta_0) \sim \mathcal{N}(0, I)$ with I being a $p \times p$ Identity matrix; $\tilde{z} \sim \mathcal{N}(0, I)$ is a random vector that is an algebraic function of z; and $\rho \geq 0$ is an arbitrary constant. Notice that the inequality (7c) follows from the Markov's bound (i.e., $\mathbf{E}[v] = \alpha \mathbf{Pr}[v \geq \alpha]$ for a nonnegative random variable v and a constant $\alpha \geq 0$).

As shown in (Rojas et al., 2011), the following expression holds for a scalar $\bar{z} \sim \mathcal{N}(0, 1)$

$$\mathbf{E}\left[\exp\left(\rho \bar{z}^2\right)\right] = \frac{1}{\sqrt{1 - 2\rho}}, \quad \forall \rho \in \left(-\infty, \frac{1}{2}\right). \qquad (8)$$

Now, (7c) and (8) can be combined to obtain

$$\mathbf{Pr}\left[\frac{1}{2}(\hat{\theta} - \theta_0)^\top V''_{app}(\theta)(\hat{\theta} - \theta_0) \geq \frac{1}{\gamma}\right]$$

$$\leq \exp\left(-\frac{2}{\gamma\rho}\right) \frac{1}{\sqrt{\det\left(I - \frac{2}{\rho}\mathcal{P}(t)^{-\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{-\frac{1}{2}}\right)}}, \qquad (9)$$

where $\rho \in \left(2\lambda_{\max}\left(\mathcal{P}(t)^{-\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{-\frac{1}{2}}\right), \infty\right)$. From (9) the sufficient condition for the control-oriented experiment design chance constraint (6) to hold is

$$\exp\left(-\frac{2}{\gamma\rho}\right) \frac{1}{\sqrt{\det\left(I - \frac{2}{\rho}\mathcal{P}(t)^{-\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{-\frac{1}{2}}\right)}} \leq \varepsilon,$$

which can be rewritten in the convex form

$$-\frac{2}{\gamma} - \frac{\rho}{2} \ln\det\left(I - \frac{2}{\rho}\mathcal{P}(t)^{-\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{-\frac{1}{2}}\right) \leq \rho \ln \varepsilon \quad (10)$$

with $\rho > 2\lambda_{max}\left(\mathcal{P}(t)^{\frac{\top}{2}} V''_{app} \mathcal{P}(t)^{\frac{1}{2}}\right)$.

Expression (10) provides a deterministic surrogate for (6). The true parameters θ_0 will be replaced with the estimated pdf of parameters (i.e., $\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_\theta)$). Assuming that the Cramér-Rao bound holds, the parameter variance-covariance matrix $\mathcal{P}(t)$ is defined as the inverse of the Fisher information matrix $\mathcal{I}(t)$

$$\mathcal{I}(t) = \sum_{k=0}^{t} \left(\frac{\partial \bar{x}(k)}{\partial \theta}\right)^\top \left(\frac{\partial \bar{x}(k)}{\partial \theta}\right), \qquad (11)$$

where the sensitivities are given by

$$\frac{d}{dt}\frac{\partial \bar{x}(t)}{\partial \theta} = \frac{\partial f}{\partial \bar{x}}\frac{\partial \bar{x}(t)}{\partial \theta} + \frac{\partial f}{\partial \theta}.$$

3.3 Uncertainty Propagation using Polynomial Chaos

The generalized polynomial chaos framework (Xiu and Karniadakis, 2002) is used for efficient propagation of the probabilistic modeling uncertainties $\hat{\theta}$. The parametric uncertainties are defined in terms of the standard random variables $\xi \in \mathbb{R}^p$. The elements $\{\xi_j\}_{j=1}^p$ are independently distributed with known pdfs $\mathsf{P}(\xi_j)$. Further, $\xi_j \in L^2\{\Omega, \mathcal{F}, \mathsf{P}\}$, where $L^2\{\cdot\}$ is the Hilbert space of ξ_j on a probability triple $(\Omega, \mathcal{F}, \mathsf{P})$ and $\mathbf{E}\left[\xi_j^2\right] < \infty$.

The gPC framework allows for approximating a stochastic variable $\psi(\xi)$ as a series expansion of polynomial basis functions

$$\psi(\xi) \approx \hat{\psi}(\xi) = \sum_{j=0}^{l} a_j \varphi_j(\xi) = a\Lambda^\top(\xi), \qquad (12)$$

where $a \triangleq [a_0, \ldots, a_l]$ are the expansion coefficients; $\Lambda(\xi) \triangleq [\varphi_0(\xi), \ldots, \varphi_l(\xi)]$ are the orthogonal basis functions from the Wiener-Askey scheme of polynomials with maximum degree m with respect to ξ; and $l+1 = \frac{(p+m)!}{p!m!}$ denotes the number of terms in the expansion. The polynomial basis functions $\varphi_j(\xi)$ are defined on the support space of ξ. The orthogonality of the basis functions implies $\langle \varphi_i(\xi), \varphi_j(\xi) \rangle = \langle \varphi_i^2(\xi) \rangle \delta_{ij} = \int_\Omega \varphi_i(\xi) \varphi_j(\xi) \mathsf{P}(\xi)\, d\xi$, where $\langle, \rangle$ denotes the inner product induced by $\mathsf{P}(\xi)$ and δ_{ij} is the Kronecker delta function. The orthogonal property of the basis functions enables efficient computation of the moments of the stochastic variable $\hat{\psi}(\xi)$ using the coefficients a.

In a stochastic model of system (1), each state $\bar{x}_i$ is approximated by a polynomial chaos expansion (12). The model equations for each approximated stochastic state $\bar{x}_i$ is written as

$$\sum_{j=0}^{l} \tilde{x}_{i,j}(t)\varphi_j(\xi) = \qquad (13)$$
$$f_i\left(\tilde{x}_1(t-1)\Lambda^\top(\xi), \ldots, \tilde{\theta}_1\Lambda^\top(\xi), \ldots, u(t-1)\right),$$

where $\tilde{x}_i$ and $\tilde{\theta}_i$ denote the coefficients of the polynomial chaos expansion for the states $\bar{x}_i$ and parameters $\hat{\theta}_i$. The coefficients $\tilde{x}_i$ and $\tilde{\theta}_i$ in (13) are determined using the Galerkin projection method (Ghanem and Spanos, 1991). The application of the Galerkin projection method to (13) yields a set of closed-form ordinary differential equations

$$\tilde{x}_i(t) = \tilde{f}_i(\tilde{x}_1(t-1), \ldots, \tilde{\theta}_1, \ldots, u(t-1)) \qquad (14)$$

for describing the dynamics of the coefficients of the expansions for each stochastic state $\bar{x}_i$.

3.4 Deterministic Surrogate for iX-SNMPC

The results presented in the preceding subsections are used to obtain a deterministic surrogate for the stochastic optimal control problem (3). The application cost function (4) is used to define the control-oriented experiment design chance constraint (6), which is approximated by (10) via Chernoff relaxation. The probabilistic uncertainties in the system model are propagated using the gPC framework, which results in the system description (14). The individual chance constraints on the states are approximated

using the Cantelli-Chebyshev inequality (Marshall and Olkin, 1979); see also (Mesbah and Streif, 2015).

The deterministic surrogate for (3) takes the form

$$\pi^* \triangleq \arg\min_\pi \; J\left[\pi, x(t_k)\right], \qquad (15a)$$

$$\text{s.t.:}\; \tilde{x}_i(t) = \tilde{f}_i\left(\tilde{x}_1(t-1), \ldots, \tilde{\theta}_1, \ldots, u(t-1)\right), \qquad (15b)$$
$$\forall t \in [t_k \;\; t_{k+N}], \quad \forall i = 1, \ldots, n$$

$$u(t) \in \mathbb{U}, \quad \forall t \in [t_k \;\; t_{k+N}] \qquad (15c)$$

$$c_i \mathbf{E}\left[\hat{x}_i\right] \le d_i - h_i \sqrt{\mathbf{Var}\left[\hat{x}_i\right]\left(\frac{1-\beta_i}{\beta_i}\right)}, \qquad (15d)$$
$$\forall t \in [t_k \;\; t_{k+N}], \quad \forall i = 1, \ldots, n$$

$$-\frac{2}{\gamma} - \frac{\rho}{2}\ln\det\left(I - \frac{2}{\rho}\mathcal{P}(t_{k+N})^{-\frac{\top}{2}}V_{\text{app}}''\right) \qquad (15e)$$
$$\mathcal{P}(t_{k+N})^{-\frac{1}{2}}\right) \le \rho\ln\varepsilon$$

$$\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_\theta) \qquad (15f)$$
$$\bar{x}(t_k) = x(t_k), \qquad (15g)$$

where $\hat{x}_i$ denotes the polynomial chaos approximation of the ith state. At every sampling time $t = t_k$, the optimal control problem (15) is solved over the prediction horizon $[t_k \;\; t_{k+N}]$, and only the first set of the optimal inputs, $u^*(t_k)$, is applied to system (1).

The receding-horizon implementation of the iX-SNMPC approach requires the knowledge of the pdf of the model parameters (i.e., $\hat{\theta} \sim \mathcal{N}(\theta_0, \mathcal{P}_\theta)$) at every sampling time t_k. Further, the system model should be adapted based on the estimated pdf of parameters at every t_k to improve the accuracy of the model predictions in the controller. To this end, the iX-SNMPC approach is implemented in conjunction with a gPC-based histogram filter (gPC HF) (Bavdekar and Mesbah, 2016), which computes the parameter pdfs conditioned on the measured system states. Thus, $\mathcal{N}(\theta_0, \mathcal{P}_\theta)$ is in fact the posterior pdf of $\hat{\theta}$ obtained using the gPC HF at every sampling instant t_k.

4. CASE STUDY: A CONTINUOUS BIOREACTOR

The proposed iX-SNMPC approach is applied to a benchmark continuous bioreactor (Agrawal et al., 1989). The system dynamics are described by

$$\dot{X} = -DX + \mu X$$
$$\dot{S} = D(S_f - S) - \frac{1}{Y_{X|S}}X$$
$$\dot{P} = -DP + (\alpha\mu + \beta)X,$$

where X is the biomass concentration; S is the substrate concentration; P is the product concentration; the dilution rate D and the inlet substrate concentration S_f are the manipulated inputs; $Y_{X|S}$ is the cell biomass yield; α and β are the yield parameters for P; and μ denotes the specific growth rate of the biomass

$$\mu = \frac{\mu_m\left(1 - \frac{P}{P_m}\right)S}{K_m + S + \frac{S^2}{K_i}}$$

with μ_m being the maximum specific growth rate. The system parameters are given in (Agrawal et al., 1989). The system states are measured at regular sampling intervals of $t = 0.25$ hr, and are corrupted by zero-mean Gaussian

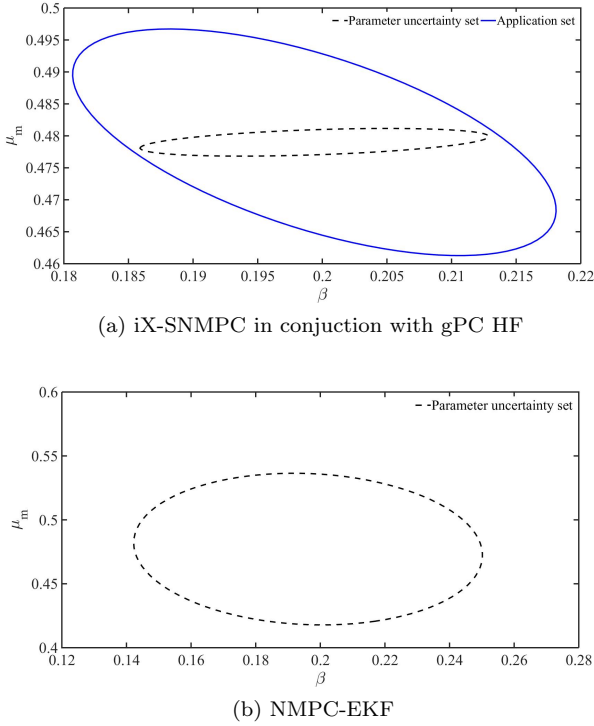(a) iX-SNMPC in conjuction with gPC HF

(b) NMPC-EKF

Fig. 1. The control-oriented application set and parameter uncertainty set at $t = 5$ hr. The model parameters are estimated based on the closed-loop data when the iX-SNMPC approach in conjunction with the gPC histogram filter and the EKF-NMPC approach are used.

measurement noise with covariance R. The parameters μ_m, $Y_{X|S}$, and β have a normal distribution, as given in Table 1. The initial conditions of the system are $x(0) = [7.038\ 2.404\ 24.87]^\top$.

The control objective in the iX-SNMPC approach is to regulate the process around a desired product concentration $\bar{P}$, while ensuring that the system model guarantees adequate control performance. The cost function is defined as

$$J = \sum_{t=t_k}^{t_{k+N}} \left(\mathbf{E}\left[P(t)\right] - \bar{P} \right)^2 + \omega \mathbf{Var}\left[P(t)\right],$$

where $\omega = 0.4$ is a constant weight. The prediction horizon and control horizon are equal and chosen as 6 hr. Hard constraints are imposed on the inputs, i.e., $0.013 \leq D \leq 0.64$ and $0.5 \leq S_f \leq 50$. In addition, product concentration P should remain below a threshold

$$\mathbf{Pr}\left[P > 26.5\right] < 0.1.$$

In the control-oriented experiment design chance constraint (2), the application bound and the admissible probability of constraint violation are defined as $\gamma = 100$ and $\varepsilon = 0.05$, respectively. The pdfs of the uncertain model

Table 1. Bioreactor system uncertainties

| Variable	pdf
μ_m	$\mathcal{N}\left(0.479, 8.1 \times 10^{-5}\right)$
$Y_{X|S}$	$\mathcal{N}\left(0.407, 8.1 \times 10^{-5}\right)$
β	$\mathcal{N}\left(0.199, 8.1 \times 10^{-5}\right)$
R	$10^{-6} \times \mathrm{diag}[1\ 30.25\ 25]$

parameters are updated using the gPC HF at every sampling time. The performance of the iX-SNMPC approach is compared to that of a nonlinear model predictive control (NMPC) approach combined with extended Kalman filter (EKF) for parameter estimation. The experiment design constraint is not included in the EKF-NMPC approach. Identical settings are considered for the two MPC approaches.

Fig. 1a shows the uncertainty set associated with the parameters β and μ_m at $t = 5$ hr. The parameters are identified based on the closed-loop data obtained when the iX-SNMPC approach is used. The parameter uncertainty set lies in the control-oriented application set. This indicates that the identified model guarantees the attainment of an admissible control performance with the least probability level 95 %. The latter (user-specified) probability level is associated with the parameter uncertainty set, and represents the least probability that the true parameters θ_0 lie in the uncertainty set. The fact that the iX-SNMPC approach ensures satisfactory control-oriented model identification in the presence of probabilistic model uncertainty results from inclusion of the experiment design chance constraint (2) into the stochastic optimal control problem. It was observed that the application set shrinks over time when the iX-SNMPC is used (not shown here). Fig. 1b shows the parameter uncertainty set when the EKF-NMPC approach is used. The size of the parameter uncertainty set is greater than that obtained using the proposed iX-SNMPC. This is attributed to model uncertainty since the control inputs in the EKF-NMPC approach are not tasked to excite the system for system identification and, consequently model uncertainty reduction.

The evolution of the estimated pdfs of μ_m using the gPC HF in the iX-SNMPC approach is shown in Fig. 2a. The figure indicates that the parameter estimates converge to
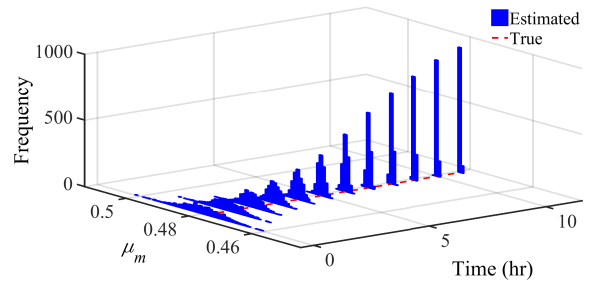
(a) iX-SNMPC approach in conjunction with gPC histogram filter
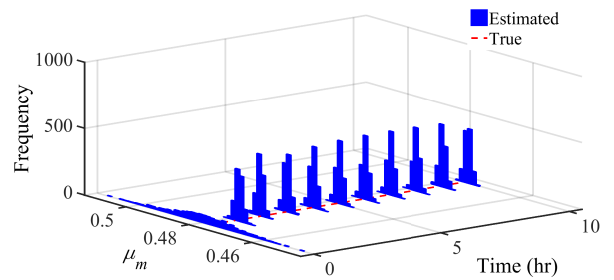
(b) EKF-NMPC approach

Fig. 2. Evolution of the pdfs of μ_m estimated based on the closed-loop data.

the true value of μ_m, while the estimated pdfs approach a Dirac-delta function after approximately $t = 6$ hr. The latter suggests reduction in the variance of the estimated parameters. In the case of the EKF-NMPC approach, there exists a small bias in the mean of the identified parameters. In contrast to the case of iX-SNMPC, the identified parameters do not monotonically converge to the true parameter value (see Fig. 2b). This results from the inability of the NMPC approach to excite the system for adequate system identification.

5. CONCLUSIONS

In this work, a stochastic model predictive control approach with integrated experiment design is presented for nonlinear systems with probabilistic parametric uncertainties. The goal of the proposed stochastic predictive control approach is twofold: (i) to enable identifying a control-oriented system model using the closed-loop data, and (ii) to regulate the system while accounting for model uncertainties. A deterministic surrogate is derived for the presented stochastic optimal control problem with control-oriented experiment design capability. The closed-loop simulation results for a continuous bioreactor benchmark demonstrate the effectiveness of the proposed approach in terms of control-oriented model adaptation using closed-loop data.

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